Tentamen Quantum Fysica I
March 7, 2000

Please **print** *your name, student number and complete address on the first page. Each problem is to be answered on a* separate *page. Print your name on top of each page.*

Elke opgave op en apart vel. Zet op het eerste vel duidelijk uw naam, student nummer en adres. Op elk volgend vel uw naam vermelden.

Problem 1.

(a) What are the operators corresponding to the momentum and energy of a particle?

(b) Write down the time-dependent Schrödinger Equation. How and under what circumstances can one derive the time-independent Schrödinger Equation from it?

(c) What is meant by a Hermitian operator? What properties do its eigenvalues and eigenfunctions have?

(d) Which of the following pairs of operators can have simultaneous eigenfunctions? Explain your answer (no more than 10 words needed).

(i) p and $T = p^2/2m$

(ii) p and $V(x)$

Problem 2.

A particle of mass m moving in one dimension is subject to an attractive delta-function potential $V(x) = V_0\,\delta(x)$ centered at the origin ($V_0 < 0$). It will turn out that there is only one bound state.

(a) Determine energy and wave function of the bound state.

(b) What is the probability for locating the particle at a distance x from the origin?

Problem 3.

(a) The operator A does not depend explicitly on time. Show that for any solution $\Psi(x,t)$ of the time-dependent Schrödinger equation

$$i\hbar \frac{d}{dt} < \Psi | A | \Psi > \;=\; < \Psi | [A, H] | \Psi > .$$

(b) Apply the result of part (a) to the momentum operator, and evaluate

$$\frac{d}{dt} < p > \;=\; ?$$

(c) The result of part (b) is known as Ehrenfest's theorem. State in words the relation this gives between quantum and classical mechanics.

Problem 4.

A particle moving in the one-dimensional square-well potential

$$V(x) = \begin{cases} 0 & x < a \\ \infty & |x| > a \end{cases}$$

is in the state

$$\psi(x) = \frac{1}{\sqrt{2}}[\phi_1(x) + \phi_2(x)]$$

at time $t = 0$, where the $\phi_n(x)$ are the normalized wave functions

$$\phi_n(x) = \frac{1}{\sqrt{a}}\cos(\frac{n\pi x}{2a}), \qquad n = 1, 3, \ldots$$

$$\phi_n(x) = \frac{1}{\sqrt{a}}\sin(\frac{n\pi x}{2a}), \qquad n = 2, 4, \ldots$$

(a) Determine E_n.

(b) What is its wave function $\psi(x,t)$ at time t?

(c) Calculate the probabilities $P_+(t)$ and $P_-(t)$ that at time t the particle is in the intervals $0 < x < a$ and $-a < x < 0$ respectively.

$$2\sin\alpha\sin\beta = \cos(\alpha - \beta) - \cos(\alpha + \beta)$$
$$2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$$
$$2\sin\alpha\cos\beta = \sin(\alpha - \beta) + \sin(\alpha + \beta)$$

(d) Interpret the time-dependence of these probabilities.

Problem 5.

The initial state $|\psi_i>$ of a quantum system is given in an orthonormal basis of three states $|\alpha>$, $|\beta>$, and $|\gamma>$ that form a complete set:

$$<\alpha|\psi_i> = i/\sqrt{3}, \quad <\beta|\psi_i> = \sqrt{2/3}, \quad <\gamma|\psi_i> = 0$$

Calculate the probability of finding the system in a state $|\psi_f>$ given in the same basis as

$$<\alpha|\psi_f> = (1+i)/\sqrt{3}, \quad <\beta|\psi_f> = \sqrt{1/6}, \quad <\gamma|\psi_f> = \sqrt{1/6}.$$